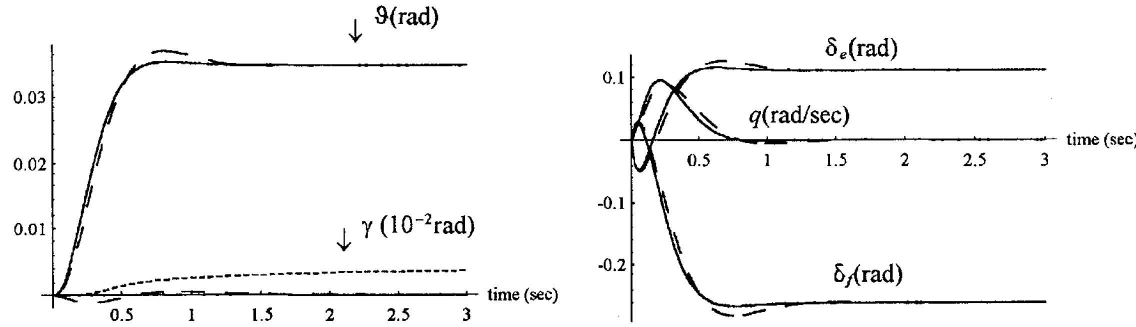

Fig. 1 Responses for pitch pointing: ——, nominal case; - - - -, nonlinear case; and — —, perturbed (uncertain) case.

To check the performance of the controller via nonlinear simulation, we substitute $Q = q + q_0$, $\dot Q = \dot q + \dot q_0$, $Z = \delta Z + Z_0$, $M = \delta M + M_0$, $\Theta = \vartheta + \vartheta_0$, $\Gamma = \gamma + \gamma_0$, $W = w + W_0$, and $\dot W = \dot w + \dot W_0$ in Eq. (1). Then Eq. (1) can be approximated for a steady flight (constant altitude and velocity) by the following nonlinear (for ϑ and γ) model:

$$\tilde{Z}_w\left[\tan(\vartheta-\gamma+\vartheta_0-\gamma_0)-\frac{W_0}{U_0}\right]+\frac{\tilde{Z}_q+U_0}{U_0}q + \frac{\tilde{Z}_{\delta_e}}{U_0}\delta_e+\frac{\tilde{Z}_{\delta_f}}{U_0}\delta_f-\frac{\tilde{g}[\cos(\vartheta_0)-\cos(\vartheta+\vartheta_0)]}{U_0} = \frac{\dot\vartheta-\dot\gamma}{\cos^2(\vartheta-\gamma+\vartheta_0-\gamma_0)} \tag{12}$$

$$\tilde{M}_w[-W_0+U_0\tan(\vartheta-\gamma+\vartheta_0-\gamma_0)]+\tilde{M}_q q - M_{\dot w}\tilde{g}[\cos(\vartheta_0)-\cos(\vartheta+\vartheta_0)]+\tilde{M}_{\delta_e}\delta_e+\tilde{M}_{\delta_f}\delta_f=\dot q$$
$$\dot\vartheta = q \tag{13}$$

where the approximations $\delta Z/m = Z_w w + Z_q q + Z_{\delta_e}\delta_e + Z_{\delta_f}\delta_f + Z_{\dot w}\dot w$ and $\delta M/I_y = M_w w + M_q q + M_{\delta_e}\delta_e + M_{\delta_f}\delta_f + M_{\dot w}\dot w$ and the assumption that the actuators are described by linear models have been used. For deriving simulation results, the trim points (initial conditions) of the model for steady flight are considered to be $\vartheta_0 = \gamma_0 = 0$ and $W_0 = 0$.

To the nonlinear system [Eqs. (12) and (13)] apply the feedback law in Eqs. (5) and (6). The resulting closed-loop system is close to being decoupled with satisfactory performance. For pitch pointing the closed-loop performance appears to be (visually) the same with that of the linear case (Fig. 1). A slight difference of about 10^{-5} rad is observed for the flight-path angle.

VII. Conclusions

The pitch and flight-path angle of a multimode aircraft have been independently controlled, via static state feedback, using the input–output decoupling with simultaneous arbitrary pole assignment technique. The satisfactory closed-loop performance has been illustrated by simulation for an AFTI F-16 aircraft. However, the present technique basically differs from that of eigenstructure assignment on the starting point, which for the present case is exact decoupling and not specification of the desired eigenvalues. Based on this difference the following results have been derived: the set of stability derivatives for which exact decoupling is satisfied (Theorem 1), the set of all decoupling controllers (in terms of stability derivatives and free parameters), and the decoupled closed-loop transfer function with arbitrary poles and gains. After appropriate evaluation of these free elements, the desirable damping and settling time are precisely obtained.

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Global Stabilization of Flexible Multibody Spacecraft Using Quaternion-Based Nonlinear Control Law

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Introduction

FLEXIBLE multibody space systems are characterized by highly nonlinear dynamics, significant elastic motion with low inherent damping, and uncertainties in the mathematical model. Global asymptotic stability of multibody flexible space systems controlled by a nonlinear dissipative controller was recently established.[1] However, the controller was restricted to have a scalar gain for quaternion feedback. Subsequently, robust nonlinear attitude control of a rigid spacecraft with nonlinear rotational dynamics was investigated using a quaternion feedback control law with a more general structure.[2] The objective of this Note is to generalize the results of Ref. 1 to include a broader class of nonlinear controllers. It can also be considered as the generalization of Ref. 2 from single-body rigid

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systems to multibody flexible systems. Furthermore, this control law is shown to provide significantly superior performance compared with that presented in Ref. 1.

Mathematical Model

We consider a nonlinear rotational dynamic model of a multibody flexible spacecraft[1] that is assumed to have a branched geometry, i.e., a central flexible body to which various flexible appendage bodies are attached. Each branch by itself could be a serial multibody structure. For simplicity, we consider a configuration in which the central body has three rigid rotational degrees of freedom and each link is connected by one rotational degree of freedom to the neighboring link. The equations of motion for such systems can be shown to have the following form[3]:

$$M(p)\ddot{p} + C(p, \dot{p})\dot{p} + D\dot{p} + Kp = B^T u \tag{1}$$

where $\dot{p} = (\omega^T, \dot{\theta}^T, \dot{q}^T)^T$; ω is the 3×1 inertial angular velocity vector (in body-fixed coordinates) for the central body; $\theta = [\theta_1, \theta_2, \ldots, \theta_{(k-3)}]^T$, where θ_i denotes the joint angle for the ith joint expressed in body-fixed coordinates; q is an $(n-k)$-dimensional vector of flexible degrees of freedom (modal amplitudes); $M(p) = M^T(p) > 0$ is the configuration-dependent mass-inertia matrix; K is the symmetric, positive semidefinite stiffness matrix; $C(p, \dot{p})$ corresponds to Coriolis and centrifugal forces; D is the symmetric, positive semidefinite damping matrix; $\{p\} = \{\varsigma^T, \theta^T, q^T\}^T$ and $\dot{\varsigma} = \omega$; and $B = [I_{k\times k} \;\; 0_{k\times(n-k)}]$ is the control influence matrix and u is the k vector of applied torques. The first three components of u represent the attitude control torques (about the x, y, z axes) applied to the central body, and the remaining components are the torques applied at the $(k-3)$ joints. K and D are given by

$$K = \mathrm{diag}\left[0_{k\times k}, \tilde{K}_{(n-k)\times(n-k)}\right]$$
$$D = \mathrm{diag}\left[0_{k\times k}, \tilde{D}_{(n-k)\times(n-k)}\right] \tag{2}$$

where $\tilde{K}$ and $\tilde{D}$ are symmetric matrices corresponding to the stiffness and damping terms of the flexible degrees of freedom. $\tilde{K}$ is inherently positive definite, and $\tilde{D}$ is assumed to be positive definite. This assumption is realistic because all physical systems have some inherent damping. The angular measurements for the central body are the three Euler angles (not the vector ς), whereas the remaining angular measurements consist of the relative angles between adjoining bodies. One important inherent property of such systems is that the matrix $(\frac{1}{2}\dot{M} - C)$ is skew symmetric (see Refs. 1 and 3).

The attitude (Euler angle) vector η of the central body is given by $E(\eta)\dot{\eta} = \omega$, where $E(\eta)$ is a 3×3 transformation matrix.[4] The sensor outputs consist of three central body Euler angles, the $(k-3)$ joint angles and the angular rates; i.e., the sensors are collocated with the torque actuators. The sensor outputs are then given by

$$y_p = B\hat{p} \quad \text{and} \quad y_r = B\dot{p} \tag{3}$$

where $\hat{p} = (\eta^T, \theta^T, q^T)^T$, wherein η is the Euler angle vector for the central body. $y_p = (\eta^T, \theta^T)^T$ and $y_r = (\omega^T, \dot{\theta}^T)^T$ are measured angular position and rate vectors, respectively. The body rate measurements ω are assumed to be available via rate gyros.

To overcome the difficulties caused by $E(\eta)$ becoming singular for some values of η, the quaternion formulation is used. The unit quaternion α is defined as follows:

$$\alpha = (\bar{\alpha}^T, \alpha_4)^T, \qquad \bar{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)^T \sin(\phi/2)$$
$$\alpha_4 = \cos(\phi/2) \tag{4}$$

where $\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)^T$ is the unit vector along the eigenaxis of rotation and ϕ is the magnitude of rotation. The quaternion also obeys the following equations[4]:

$$\bar{\alpha}^T\bar{\alpha} + \alpha_4^2 = 1 \tag{5}$$

$$\dot{\bar{\alpha}} = \tfrac{1}{2}(\omega \times \bar{\alpha} + \alpha_4\omega) \qquad \dot{\alpha}_4 = -\tfrac{1}{2}\omega^T\bar{\alpha} \tag{6}$$

The quaternion representation will be used for the central body attitude. Defining $\beta = (\alpha_4 - 1)$ and denoting $\dot{p} = z$, Eqs. (1) and (6) can be rewritten as follows:

$$M\dot{z} + Cz + Dz + K\left[0_{1\times k}\, q^T\right]^T = B^T u \tag{7}$$

$$(\dot{\theta}^T, \dot{q}^T)^T = \left[0_{(n-3)\times 3}, I_{(n-3)\times(n-3)}\right]z \tag{8}$$

$$\dot{\bar{\alpha}} = \tfrac{1}{2}[\omega \times \bar{\alpha} + (\beta + 1)\omega]; \qquad \dot{\beta} = -\tfrac{1}{2}\omega^T\bar{\alpha} \tag{9}$$

As discussed in Ref. 1, the system represented by Eqs. (7–9) can be expressed in the state–space form as follows:

$$\dot{x} = f(x, u) \tag{10}$$

where $x = (\bar{\alpha}^T, \beta, \theta^T, q^T, z^T)^T$. The dimension of x is $(2n+1)$, which is one more than the dimension of the system in (1) with one constraint [(Eq. (5)]. A nonlinear control law, which is a generalization of the results of Ref. 1 and which extends the results of Ref. 2 to the multibody flexible system case, is given next.

Nonlinear Dissipative Control Law

Consider the control law u, given by

$$u = -G_p\tilde{p} - G_r y_r \tag{11a}$$

where

$$G_p = \mathrm{diag}\left\{\tfrac{1}{2}[(\tilde{\bar{\alpha}} + I_3\alpha_4)G_{p1} + \gamma(1 - \alpha_4)I_3], G_{p2}\right\} \tag{11b}$$

Matrices G_p and G_r are symmetric positive definite $(k \times k)$ matrices and $\tilde{p} = (\bar{\alpha}^T, \theta^T)^T$. γ is a positive scalar; $\tilde{\bar{\alpha}}$ represents a cross-product operator matrix $(\bar{\alpha}\times)$, and G_{p1}, G_{p2} are (3×3) and $(k-3)\times(k-3)$ symmetric positive definite matrices, respectively. Note that the feedback $\tilde{p}$ can be computed using the sensor measurements y_p using quaternion equations.[4] The following Lemma gives the closed-loop equilibrium solutions. This Lemma is an extension of Lemma 1 in Ref. 2 to multibody flexible spacecraft.

Lemma 1: Suppose G_{p1} and G_{p2} are symmetric and positive definite and $0 < \lambda_M(G_{p1}) \le 2\gamma$, where $\lambda_M(\cdot)$ denotes the largest eigenvalue. Then the closed-loop system given by (10) and (11) has exactly two equilibrium solutions: $[\tilde{p} = q = \dot{p} = 0, \beta = 0]$ and $[\tilde{p} = q = \dot{p} = 0, \beta = -2]$.

Proof: The closed-loop equilibrium solutions can be obtained by equating all time derivatives in Eq. (10) to zero, i.e., $\dot{p} = \ddot{p} = 0 \Rightarrow \omega = 0, \dot{\theta} = 0$, and $\dot{q} = 0$. Substituting in Eq. (7), we get

$$-B^T G_p\tilde{p} = \begin{bmatrix} -\frac{1}{2}\{[\tilde{\bar{\alpha}} + I_3(\beta+1)]G_{p1} - \gamma\beta I_3\}\bar{\alpha} \\ -G_{p2}\theta \\ 0_{(n-k-3)\times 1} \end{bmatrix} = \begin{bmatrix} 0_{3\times1} \\ 0_{k\times1} \\ \tilde{K}q \end{bmatrix} \tag{12}$$

$$\Rightarrow \tfrac{1}{2}\{[\tilde{\bar{\alpha}} + I_3(\beta+1)]G_{p1} - \gamma\beta I_3\}\bar{\alpha} = 0,$$
$$\theta = 0, \qquad q = 0 \tag{13}$$

Premultiplying the first of the three Eqs. (13) by $\bar{\alpha}^T$, we get

$$\bar{\alpha}^T\mathcal{M}\bar{\alpha} = 0, \quad \text{where } \mathcal{M} = (\beta+1)G_{p1} - \gamma\beta I_3 \tag{14}$$

The eigenvalues of $\mathcal{M}$ are given by

$$\lambda_i(\mathcal{M}) = (\beta+1)\lambda_i(G_{p1}) - \gamma\beta = (\beta+1)[\lambda_i(G_{p1}) - \gamma] + \gamma$$

It can be easily proved that for the cases when $0 < \lambda_i(G_{p1}) < \gamma$, $\lambda_i(G_{p1}) = \gamma$ and $\gamma < \lambda_i(G_{p1}) < 2\gamma$, matrix $\mathcal{M}$ is nonsingular for any feasible values of $\bar{\alpha}$. For $\lambda_i(G_{p1}) = 2\gamma$, $\mathcal{M}$ is singular only when $\beta = -2$, i.e., when $\bar{\alpha} = 0$. For the case when $\lambda_i(G_{p1}) > 2\gamma$, there are feasible nonzero solutions of $\bar{\alpha}$ for which $\mathcal{M}$ could become singular. Hence, for $\lambda_i(G_{p1}) \le 2\gamma$ the closed-loop system has only two possible equilibrium solutions: $[\tilde{p} = q = \dot{p} = 0, \beta = 0]$ and $[\tilde{p} = q = \dot{p} = 0, \beta = -2]$. □

Thus, there appear to be two closed-loop equilibrium points corresponding to $\beta = 0$ and $\beta = -2$ (all other state variables being zero).

However, from Eq. (4), $\beta = 0 \Rightarrow \phi = 0$ and $\beta = -2 \Rightarrow \phi = 2n\pi$, i.e., there is only one equilibrium point in the physical space. The global asymptotic stability of the physical equilibrium state (the origin of the state space) is proved next.

Theorem 1: Suppose G_{p1}, G_{p2}, and G_r are symmetric and positive definite and $0 < \lambda_M(G_{p1}) \leq 2\gamma$. Then, the closed-loop system given by Eqs. (10) and (11) is globally asymptotically stable.

Proof: Consider the candidate Lyapunov function

$$V = \tfrac{1}{2}\dot{p}^T M(p)\dot{p} + \tfrac{1}{2}q^T \tilde{K} q + \tfrac{1}{2}\theta^T G_{p2}\theta + \tfrac{1}{2}\bar{\alpha}^T G_{p1}\bar{\alpha} + \tfrac{1}{2}\gamma\beta^2 \quad (15)$$

Taking the time derivative of V, evaluating $\dot{V}$ along system trajectories (1), using Lemma 1 and Eq. (9), and simplifying and rearranging the terms, we get

$$\dot{V} = -\dot{q}^T \tilde{D}\dot{q} + (B\dot{p})^T u + \dot{\theta}^T G_{p2}\theta + (\omega^T/2)[\tilde{\bar{\alpha}} G_{p1} + (\beta+1)G_{p1} - \gamma\beta]\bar{\alpha} \quad (16)$$

Using control law (11) in Eq. (16) yields

$$\dot{V} = -\dot{q}^T \tilde{D}\dot{q} - y_r^T G_r y_r = -\dot{p}^T\left(D + B^T G_r B\right)\dot{p} \quad (17)$$

Because $(D + B^T G_r B)$ is a positive definite symmetric matrix, $\dot{V} \leq 0$, i.e., $\dot{V}$ is negative semidefinite. Also, $\dot{V} = 0 \Rightarrow \dot{p} = 0 \Rightarrow \ddot{p} = 0$. Substituting in the closed-loop equation and using Lemma 1 gives two equilibrium solutions: $[\bar{p} = q = \dot{p} = 0, \beta = 0]$ and $[\bar{p} = q = \dot{p} = 0, \beta = -2]$. However, as stated previously, these two equilibrium points correspond to the same physical equilibrium state.[2] Hence, it is shown that V is negative along all system trajectories except at the two equilibrium points that represent the same physical equilibrium state, and therefore, by LaSalle's invariance theorem (as used in Ref. 2) it can be concluded that the system is globally asymptotically stable. □

It can be verified that the scalar-gain control law of Ref. 1 is a special case of the control law in Eq. (12) obtained by setting $G_{p1} = 3\mu I_3$ and $\gamma = 2\mu$.

Numerical Example

Consider the multibody flexible spacecraft (Fig. 1) that resembles a space robot. For the purpose of comparison, the example system used here is the same as that in Ref. 1. The system consists of a rigid base body attached to which are two articulated flexible links. For comparison, the same rest-to-rest maneuver as the one used in Ref. 1 was considered with initial orientation equivalent to a rotation of the entire spacecraft by $\pi/4$ rad about the global X axis and 0.5 rad rotation of revolute joint 2. The control law (11) was used to restore the spacecraft's zero-state configuration.[1] The control gains were chosen based on several trials because no systematic synthesis technique exists. A suitable choice of gains was found to be G_{p1} = diag[550, 425, 350], G_{p2} = diag[60, 70], G_r = diag[450, 275, 300, 80, 90], and $\gamma = 1000$. Figure 2 shows the comparison of responses for Euler parameter 1, torque for Euler axis 1, and displacement of revolute joint 2 for two different control laws. (The remaining responses had no significant differences and are not shown.) Note that, although the position gains used here for the central body attitude (G_{p1}) are almost half of those used in Ref. 1, the response has improved, resulting in less overshoot for the same settling time. This improvement can be attributed to the additional terms and the additional design freedom available in the choice of G_{p1} and γ with the new control law. As a consequence, the control torque for Euler axis 1 has been significantly reduced, as shown in Fig. 2b. A similar improvement was observed in the displacement response of revolute joint 2 (Fig. 2c). The overall saving achieved in the control energy

$$= \int_0^{40} u^T u \, dt$$

was 40% compared with Ref. 1, along with significant improvement in the responses. The response for the y displacement of the link 2 tip did not show any significant differences between the two control laws.

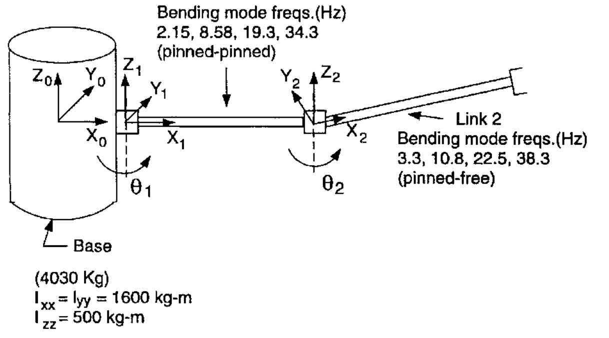

Fig. 1 Multibody flexible spacecraft.

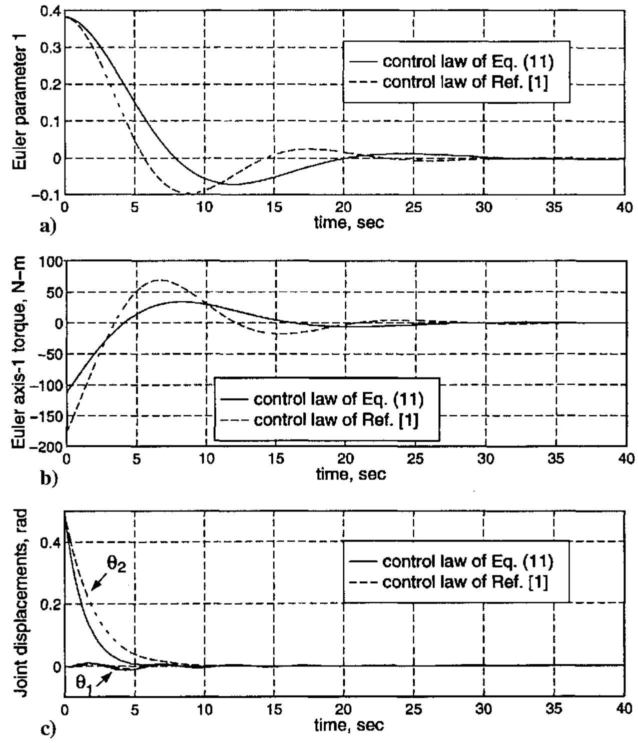

Fig. 2 Comparison of responses for two control laws.

Concluding Remarks

A nonlinear quaternion-based feedback control law was shown to provide global asymptotic stability in large-angle maneuvers of multibody flexible spacecraft. This control law generalizes the previous results on a quaternion-based scalar-gain control law for multibody spacecraft as well as a quaternion-based nonlinear control law for single-body rigid spacecraft. The simulation results presented demonstrate that this control law can provide superior performance with reduced control energy requirement, as compared with the scalar-gain control law. The control law is also robust to modeling errors and parametric uncertainties because it does not depend on the knowledge of the system parameters.

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